Greeks

We use finite difference method and pathwise derivative method to estimate the Greeks.

1.

The approach for finite difference is the central difference estimator. Suppose the estimated price is given by

,

whereis the number of simulation, is the simulated samples, andis the parameter of interest. Letbe the step size we want to add to or subtract from the original, then we can form the central difference estimator:

for first-order derivatives,



for second-order derivatives,



2.

The idea of pathwise differentiation is also known as infinitesimal perturbation analysis (IPA). The basis is to compute the derivative of the sample discounted payoffs with respect to the parameter of interest. Suppose the price is given by

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so the sensitivity is

.

The question of whether exchanging the operations of expectation and differentiation is justified is discuss in [] []. And it turns out that this is applicable for European options.

For European call option, the payoff function is given by



We need to compute the derivatives for each simulated sample quantity. [] has already give some basic formulas of derivatives. So the corresponding sensitivities can be derived as,









where,





For gamma, the second-order derivative, we can use pathwise differentiation to compute the deltas first, and then use central difference method to differentiate the delta. That is,



3.

Based on the mathematics above, we calculates the delta, gamma, and rho of European call option with the parameters set at the very beginning. The results are in table\_\_\_\_.

|  |  |  |
| --- | --- | --- |
|  | European Call Greek Value | Standard Error |
| Delta diff | 0.47934777 | 0.03566071 |
| Delta pathwise | 0.465494395 | 0.001945311 |
| Delta fourier | 0.4653853 | NA |
| Gamma diff | 0.09000361 | 0.12400942 |
| Gamma pathwise | 0.018325135 | 0.001374918 |
| Gamma fourier | 0.01978011 | NA |
| Rho diff | 39.6033748 | 0.3656007 |
| Rho pathwise | 38.9640694 | 0.1599638 |
| Rho fourier | 39.13612 | NA |

The Greeks obtained by the two methods described above are compare with the bench mark Greeks, which are calculated with Fourier method. The results of pathwise derivative are very close to the bench mark value, but finite difference results in larger deviations. It’s very clear that the standard error of finite difference is much higher than that of pathwise. Besides, the selection of step size in finite difference method should also be taken into account for the variance issue. Therefore, it indicates that pathwise derivative performs better than finite difference.

We also use pathwise derivative method to investigate how the Greeks are changing with the VG parameters. The figure \_\_ shows that when the volatility is increasing, the sensitivities of option price with respect to underlying asset and volatility is also going upward. The sensitivity to interest rate goes up fast at the beginning and then slowly goes downward. We notice that there is a kind of threshold in the plots of delta, rho and vega when sigma is around 0.1. Their trends behave differently when sigma is less or greater than 0.1. In figure \_\_, delta, gamma and rho appear to be negatively related to theta. Vega seems to reach its highest value when theta is zero, which means the distribution is symmetric. The plots in figure are much clearer in their trends. The delta, rho and vega decline as the kurtosis parameter goes up, but gamma is positively related to nu.





